RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 You are given that matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and matrix $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$.

(i) Find **BA**. [2]

(ii) A plane shape of area 3 square units is transformed using matrix **A**. The image is transformed using matrix **B**. What is the area of the resulting shape? [3]

2 You are given that $\alpha = -3 + 4\mathrm{j}$.

(i) Calculate α^2. [2]

(ii) Express α in modulus-argument form. [3]

3 **(i)** Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. [5]

(ii) Show the roots on an Argand diagram. [2]

4 Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n} [(r+1)(r-2)] = \frac{1}{3}n(n^2 - 7)$. [6]

5 The equation $x^3 + px^2 + qx + r = 0$ has roots α, β and γ, where

$$\alpha + \beta + \gamma = 3,$$
$$\alpha\beta\gamma = -7,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 13.$$

(i) Write down the values of p and r. [2]

(ii) Find the value of q. [3]

6 A sequence is defined by $a_1 = 7$ and $a_{k+1} = 7a_k - 3$.

(i) Calculate the value of the third term, a_3. [2]

(ii) Prove by induction that $a_n = \dfrac{(13 \times 7^{n-1}) + 1}{2}$. [6]

Section B (36 marks)

7 The sketch below shows part of the graph of $y = \dfrac{x-1}{(x-2)(x+3)(2x+3)}$. One section of the graph has been omitted.

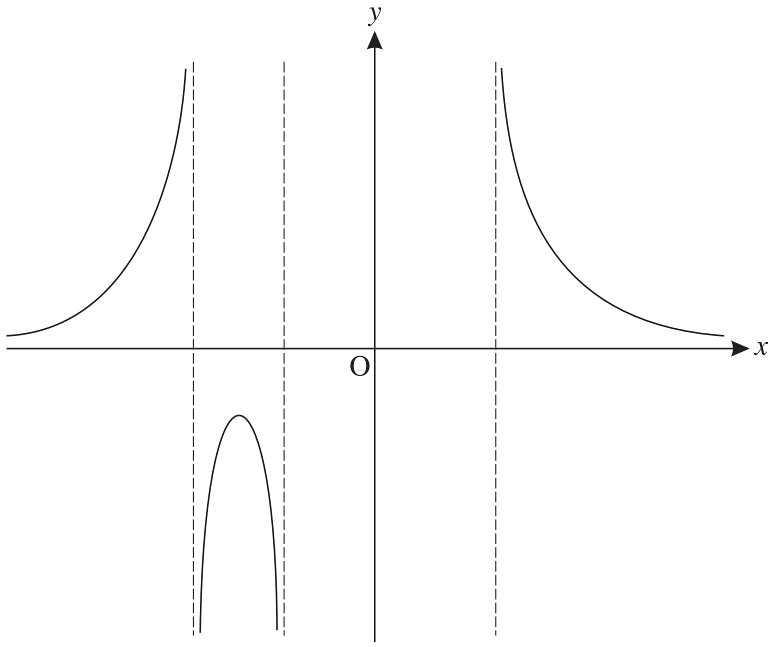

Fig. 7

(i) Find the coordinates of the points where the curve crosses the axes. [2]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Copy the sketch and draw in the missing section. [2]

(iv) Solve the inequality $\dfrac{x-1}{(x-2)(x+3)(2x+3)} \geqslant 0$. [3]

8 **(i)** On a single Argand diagram, sketch the locus of points for which

(A) $|z - 3\mathrm{j}| = 2$, [3]

(B) $\arg(z+1) = \frac{1}{4}\pi$. [3]

(ii) Indicate clearly on your Argand diagram the set of points for which

$$|z - 3\mathrm{j}| \leqslant 2 \quad \text{and} \quad \arg(z+1) \leqslant \tfrac{1}{4}\pi. \qquad [2]$$

(iii) (A) By drawing an appropriate line through the origin, indicate on your Argand diagram the point for which $|z - 3\mathrm{j}| = 2$ and $\arg z$ has its minimum possible value. [2]

(B) Calculate the value of $\arg z$ at this point. [2]

9 A transformation T acts on all points in the plane. The image of a general point P is denoted by P′. P′ always lies on the line $y = x$ and has the same x-coordinate as P. This is illustrated in Fig. 9.

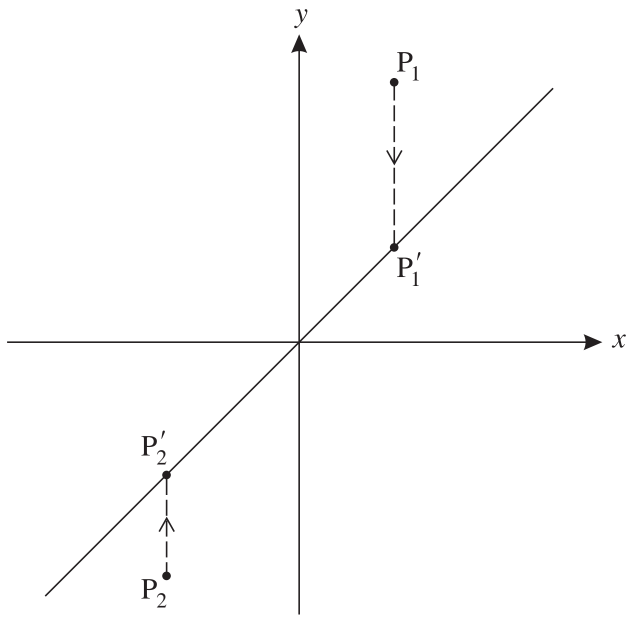

Fig. 9

(i) Write down the image of the point $(-3, 7)$ under transformation T. [1]

(ii) Write down the image of the point (x, y) under transformation T. [2]

(iii) Find the 2×2 matrix which represents the transformation. [3]

(iv) Describe the transformation M represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [2]

(v) Find the matrix representing the composite transformation of T followed by M. [2]

(vi) Find the image of the point (x, y) under this composite transformation. State the equation of the line on which all of these images lie. [3]

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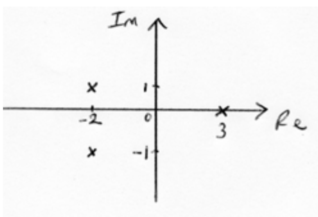
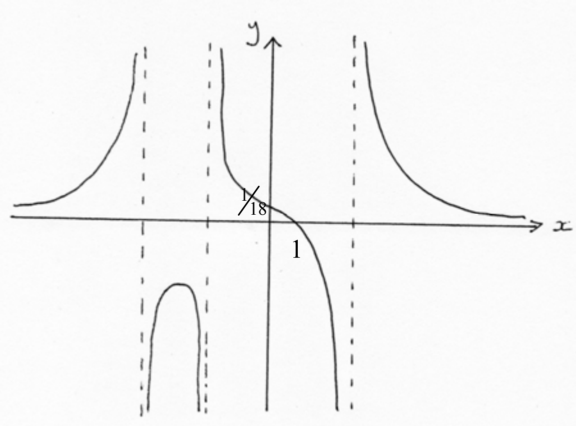
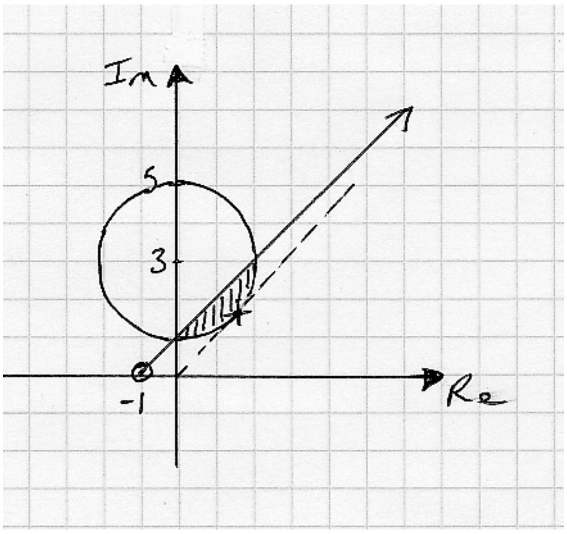
Qu	Answer	Mark	Comment
Section A			
1(i)	$\mathbf{BA}=\begin{pmatrix}3&1\\-2&4\end{pmatrix}\begin{pmatrix}2&-1\\0&3\end{pmatrix}=\begin{pmatrix}6&0\\-4&14\end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply c.a.o.
1(ii)	$\det\mathbf{BA}=(6\times14)-(-4\times0)=84$ $3\times84=252$ square units	M1 A1 A1(ft) **[3]**	Attempt to calculate any determinant c.a.o. Correct area
2(i)	$\alpha^2=(-3+4\mathrm{j})(-3+4\mathrm{j})=(-7-24\mathrm{j})$	M1 A1 **[2]**	Attempt to multiply with use of $\mathrm{j}^2=-1$ c.a.o.
2(ii)	$\lvert\alpha\rvert=5$ $\arg\alpha=\pi-\arctan\frac{4}{3}$ = 2.21 (2d.p.) (or 126.87°) $\alpha=5(\cos2.21+\mathrm{j}\sin2.21)$	B1 B1 B1(ft) **[3]**	Accept 2.2 or 127° Accept degrees and $(r,\ \theta)$ form s.c. lose 1 mark only if α^2 used throughout (ii)
3(i)	$3^3+3^2-7\times3-15=0$ $z^3+z^2-7z-15=(z-3)(z^2+4z+5)$ $z=\dfrac{-4\pm\sqrt{16-20}}{2}=-2\pm\mathrm{j}$ So other roots are $-2+\mathrm{j}$ and $-2-\mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	Showing 3 satisfies the equation (may be implied) Valid attempt to factorise Correct quadratic factor Use of quadratic formula, or other valid method One mark for both c.a.o.
3(ii)		B2 **[2]**	Minus 1 for each error ft provided conjugate imaginary roots

4	$\sum_{r=1}^{n}\left[(r+1)(r-2)\right]=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-2n$ $=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-2n$ $=\frac{1}{6}n\left[(n+1)(2n+1)-3(n+1)-12\right]$ $=\frac{1}{6}n\left(2n^2+3n+1-3n-3-12\right)$ $=\frac{1}{6}n\left(2n^2-14\right)$ $=\frac{1}{3}n\left(n^2-7\right)$	M1 A2 M1 M1 A1 **[6]**	Attempt to split sum up Minus one each error Attempt to factorise Collecting terms All correct
5(i) **5(ii)**	$p=-3,\ r=7$ $q=\alpha\beta+\alpha\gamma+\beta\gamma$ $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=(\alpha+\beta+\gamma)^2-2q$ $\Rightarrow 13=3^2-2q$ $\Rightarrow q=-2$	B2 **[2]** B1 M1 A1 **[3]**	One mark for each s.c. B1 if *b* and *d* used instead of *p* and *r* Attempt to find *q* using $\alpha^2+\beta^2+\gamma^2$ and $\alpha+\beta+\gamma$, but not $\alpha\beta\gamma$ c.a.o.
6(i) **6(ii)**	$a_2=7\times 7-3=46$ $a_3=7\times 46-3=319$ When *n* = 1, $\frac{13\times 7^0+1}{2}=7$, so true for *n* = 1 Assume true for *n* = *k* $a_k=\frac{13\times 7^{k-1}+1}{2}$ $\Rightarrow a_{k+1}=7\times\frac{13\times 7^{k-1}+1}{2}-3$ $=\frac{13\times 7^k+7}{2}-3$ $=\frac{13\times 7^k+7-6}{2}$ $=\frac{13\times 7^k+1}{2}$ But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1. Since it is true for *k* = 1, it is true for *k* = 1, 2, 3 and so true for all positive integers.	M1 A1 **[2]** B1 E1 M1 A1 E1 E1 **[6]**	Use of inductive definition c.a.o. Correct use of part (i) (may be implied) Assuming true for *k* Attempt to use $a_{k+1}=7a_k-3$ Correct simplification Dependent on A1 and previous E1 Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$(1,\ 0)$ and $(0,\ \frac{1}{18})$	B1 B1 [2]	
7(ii)	$x = 2,\ x = -3,\ x = \frac{-3}{2},\ y = 0$	B4 [4]	Minus 1 for each error
7(iii)		B1 B1 [2]	Correct approaches to vertical asymptotes Through clearly marked $(1,\ 0)$ and $(0,\ \frac{1}{18})$
7(iv)	$x < -3$, $x > 2$ $\frac{-3}{2} < x \le 1$	B1 B2 [3]	 B1 for $\frac{-3}{2} < x < 1$, or $\frac{-3}{2} \le x \le 1$
8(i) 8(ii)		B3 B3 [6] B2(ft) [2]	Circle, B1; radius 2, B1; centre 3j, B1 Half line, B1; from -1, B1; $\frac{\pi}{4}$ to x-axis, B1 Correct region between their circle and half line indicated s.c. B1 for interior of circle
	Sketch should clearly show the radius and centre of the circle and the starting point and angle of the half-line.	M1 A1(ft)	Tangent from origin to circle Correct point placed by eye where tangent from origin meets circle
8(iii)	$\arg z = \frac{\pi}{2} - \arcsin\frac{2}{3}$ = 0.84 (2d.p.)	M1 A1 [4]	Attempt to use right angled triangle c.a.o. Accept 48.20° (2d.p.)

9(i)	$(-3,\ -3)$	B1 **[1]**	
9(ii)	$(x,\ x)$	B1 B1 **[2]**	
9(iii)	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to min of 0
9(iv)	Rotation through $\frac{\pi}{2}$ anticlockwise about the origin	B1 B1 **[2]**	Rotation and angle (accept 90°) Centre and sense
9(v)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply using their **T** in correct order c.a.o.
9(vi)	$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ x \end{pmatrix}$ So (-*x*, *x*) Line *y* = -*x*	M1 A1(ft) A1 **[3]**	May be implied c.a.o. from correct matrix

4755: Further Concepts for Advanced Mathematics (FP1)

General Comments

The overall standard of the candidates was high, with the majority clearly well prepared for this paper. There were, however, a very small number of centres where the candidates scored few marks; perhaps they had been entered too early.

Although there were many high marks, relatively few candidates scored in the upper 60s or higher; certain part-questions, particularly the end of question 8, were found very difficult.

A large number of avoidable errors were made by candidates failing to read the questions sufficiently carefully.

Marks were lost by some candidates who failed to label diagrams clearly, or who showed insufficient workings in their solutions.

Comments on Individual Questions

1) **Matrices**

Usually well answered. In (i) a few multiplied **AB** instead of **BA**. In (ii) the most common error was to multiply 3 by either det **A** or det **B**, but not both. More candidates who did this correctly seemed to use det **A** and det **B** rather than det **BA**. A very few tried to transform a square or rectangle of area 3 but this method was very rarely successful.

2) **Complex numbers and modulus-argument form**

Almost all candidates got part (i) right; there were just a few careless mistakes.

In part (ii) many candidates did not obtain the correct argument of the angle; it was often the case that they did not know which angle was required. Those candidates who sketched -3+4j on an Argand diagram were nearly always successful. Other common errors were giving the modulus as 25 rather than 5, and not knowing the meaning of modulus-argument form.

3) **Complex numbers and the roots of a cubic**

Almost all candidates were successful in showing that 3 is one root of the equation and most found the other two roots, although there were a few careless mistakes.

In part (ii) the complex roots were usually shown correctly on the Argand diagram but many lost a mark by omitting the real root.

4) **Using standard results to prove the formula for the sum of a series**

Nearly all candidates knew how to answer this question but many of them made the mistake of writing $\sum 2 = 2$ instead of $\sum 2 = 2n$.

While most candidates scored fairly well on this question, a significant few seemed totally unprepared for a question of this type. A small number tried to prove the result by induction.

5) **Relationships between the roots of a cubic equation**

Most candidates knew what to do in this question but there were plenty of careless mistakes, many of them because of sign errors. Another common error was to omit the 2 in the expansion $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$.

6) **Proof by induction**

Most candidates knew what to do in part (i), applying an inductive definition, and obtained the right answer. There were, however, a few careless arithmetical mistakes.

In part (ii), candidates were asked to prove a result by induction. The question asked for proof of the formula for the general term of a sequence, rather than for the sum of the sequence. Many candidates did not appreciate this and tried to find the sum instead. However, many others did realise what was required and obtained full marks on this question.

In attempting to prove the inductive step, a few candidates made the error $7 \times 7^{k-1} = 49^{k-1}$.

Some candidates were not good at explaining the implication of their workings; 'therefore by induction the result is true for all *n*' is not sufficient to earn the final 2 marks unless the candidate convincingly explains why this is the case – see the mark scheme.

7) **Graph**

This question was very well answered. Many candidates obtained full marks, or nearly so.

(i) This was well answered, but some candidates omitted one or other of the two points, usually the intersection with the *y*-axis.

(ii) Almost all candidates scored full marks but a few either omitted the horizontal asymptote or gave an incorrect answer.

(iii) Most drew the correct shape but many lost a mark by failing to write the co-ordinates of the points of intersection of the curve with the axes on their sketches.

(iv) Most candidates obtained the required regions but many did not give strict inequalities where they were required and so lost marks.

8) **Loci on the Argand diagram**

This was the least well answered question on the paper.

(i) Many candidates ignored the instruction to draw the two loci on a single Argand diagram.

Many candidates drew the circle with centre (0, -3j) instead of (0, 3j), and the half-line beginning at (1, 0) instead of (-1, 0). Those who made this error ended up with no obvious intersection for part (ii).

(ii) In a large number of scripts the instruction regarding the inequalities was misinterpreted as "either … or…" , rather than "and". This was often the case even with correctly drawn sketches.

(iii) Few candidates made much progress on part (iii). Many of those who drew the tangent from the origin to the circle assumed it made an angle of $\frac{\pi}{4}$ with the real axis. Most failed to indicate the required point. It was only the very strongest candidates who obtained the correct answer to this part.

9) **Matrix transformations**

Many answers were fully correct. There was a very small number of candidates who worked with row vectors rather than columns, but very few of these were consistent throughout their answers.

(i) Almost all candidates got this right.

(ii) Almost all candidates got this right.

(iii) The large majority of candidates got this right.

(iv) Whilst most got this right, many gave incomplete descriptions, most commonly omitting either the centre or the direction of the rotation.

(v) **TM** was quite commonly calculated instead of **MT**.

(vi) Those who obtained the correct answer to part (v) usually went on to obtain the correct answer to part (vi).